and that it therefore does not fairly meet the requirements of Insurance Company problems, fully considered. Further, that to improve such a system, not merely methods of determining the relative prices or premiums for one contingency as compared with another must be involved, but also methods for marshalling results, whether in excess or defect, that may not be in exact accordance with the prices or premiums originally charged.

In a subsequent paper on independent risks, conjoined with a doctrine of limits, I shall at least be able, I trust, to take the first step towards showing that a system, sufficiently combining these requirements, is not, even in practice, wholly beyond our reach. In the mean time, I may be allowed to remark, that, however well the prevailing system may have on occasions appeared to have fulfilled its purpose, it has almost always only done so by including an element upon which dependence can no longer be placed. I allude to what have been called "errors on the safe side," or exaggerations of the primitive ratios. So long as such errors were allowed to remain on the so-called safe side—if errors can ever be said to be safe in any sense—so long a somewhat crude system of calculation might be allowed with some impunity. But when, in the face of competition for public favour, and under the name of the bonus system and distribution of profits, such exaggerations themselves have had, and still have, to be dealt with and distributed, any presumptions of safety that might otherwise have arisen under the plea of excess of caution are in constant danger of becoming elements of evil, by some counter process or procedure which a more comprehensive system of calculation would have rendered unnecessary, or, if necessary, would have legalized.

On the Determination of the Rates of Premium for Assuring against Issue.   By ARCHIBALD DAY, *Actuary of the London and Provincial Law Assurance Society and Fellow of the Institute of Actuaries.*

AMONG the numerous classes of risks now undertaken by Insurance Companies, those against the contingency of leaving issue are beginning, from their increasing numbers, to assume a position of greater importance than heretofore. Nothing has hitherto, to my

knowledge, been written upon the subject, and the premiums for insurances of this nature have been left to the judgment of the actuary as each individual case has come before him, there being no recognized and accurate formula for the calculation of the risk. Under these circumstances, and as a considerable amount of material exists from which approximate results may be deduced, I venture to submit to the Institute a few considerations upon the subject, but confined principally to the probabilities of marriage so far as they affect the calculation of such premiums.*

It will, I think, be generally allowed, that insurances payable upon the death of a person, in the event of his leaving issue surviving, form a very legitimate class of business. The proposer is generally the person next in succession, in the event of the life to be insured dying without issue, and his object is frequently to perfect a security in borrowing money upon his contingent reversion. A policy of this nature would hardly be taken out as a speculation by the tenant for life himself, as the sum assured not being payable until death, he would personally have no interest in it; but, at the same time, there is, of course, great room for fraud in this, as well as in almost every class of insurance transactions. Proposals, practically, seldom come before Assurance Societies unless there is a reasonable prospect that the reversioner will come into possession—arising either from the advanced age of the tenant for life, or from his having passed a long period of married life without family, or from the death of children who would otherwise have succeeded—the youngest age, in my recollection, upon which such a risk has been undertaken, being 48 years. In most instances, the life has been at the time married; but cases have occasionally occurred in which he has been either a bachelor or widower —the risk being then, of course, considerably increased.

In the observations following, the husband and wife are both assumed to be alive at the time of granting the insurance; and it is further assumed, as, in fact, is almost always the case, that the chances of issue by the existing marriage are too remote to affect the rate of premium to be charged.

* The following passage, which occurs in the article in the *Edinburgh Review* just published on Life Assurance, refers to the present subject:—

" Amongst the more curious cases occasionally brought before actuaries are those professionally called issue cases. The individual entitled to a life interest in a certain property, if another now in possession of it should die without leaving issue, may resort to a Life Office to raise money upon his contingent life interest—to effect which he must assure against the life tenants leaving issue. Sometimes such contingency is naturally very remote, but the transaction being peculiar, premiums of 20s., 30s., or even 40s. per cent. per annum have been demanded for such assurance, probably because actuaries have been unable to obtain a close approximation to the actual risk."

It would probably be found that great differences exist in the methods hitherto adopted in the calculation of the premiums; but, I believe, a rough approximation has not unfrequently been obtained from the following method :—Allowing that no issue would arise by the existing marriage, the risk would terminate in the event of the husband dying before the wife; the premium charged has therefore been that for a sum payable on the death of the husband, provided he died after the wife, and, making the assumption that it is an even chance that he would remarry and have issue by the second wife, is represented by the formula—

$$\tfrac{1}{2}A_{HW}, \text{ or } \tfrac{1}{2}(A_H - A_{HW}).$$
$$\quad\;\; {\scriptstyle(2)} \qquad\qquad\qquad {\scriptstyle(1)}$$

Probably other methods, with which I am not familiar, may be disclosed if any discussion should arise upon this paper. Inquiries having become more frequent as to the rates which would be charged for cases of this description, led me to reflect whether some method could not be found to provide for the contingency, with somewhat less of the arbitrary character of the foregoing; and the results I obtained, aided by the kind suggestions of my friend Mr. Bailey, are now offered to the Institute.

In the lucid Reports of the Registrar-General will be found tables of the number of marriages contracted in each year, in a considerable proportion of which the respective ages (within quinquennial periods) of both husband and wife are distinguished. In the year 1851, the total number of marriages that were solemnized was 154,206, and of these the ages of both parties were given, in 56,347 instances, or rather more than one-third of the whole number.

Of these there were—

Bachelors . . .	48,555	Spinsters . . .	51,141
Widowers . .	7,792	Widows . . .	5,206
	56,347		56,347

It may fairly be assumed, that, in the cases of those whose ages were not known, the numbers at each quinquennial period would bear the same proportion to the total number as in the instances where the ages of both were given.

From Table No. VII. of the Summary Tables appended to the Report of the Census Commissioners, 1851, headed, " Civil Condition of the People," we obtain the number of males and females living in that year, married and unmarried, for every five years of age, and from these two sources the following table has been constructed :—

TABLE I.

Age.	Number Living, 1851.			Actual Marriages in 1851, where ages of both husband and wife were given.			Estimated Marriages in 1851.		
	Bachelors.	Widowers.	Total Number of Males Unmarried.	Bachelors.	Widowers.	Total.	Bachelors.	Widowers.	Total.
15–	869,325	76	869,401	1,477	..	1,477	4,035	..	4,035
20–	633,842	2,168	636,010	26,008	241	26,249	71,049	667	71,716
25–	308,363	8,290	316,653	13,781	1,072	14,853	37,647	2,967	40,614
30–	159,956	14,249	174,205	4,597	1,474	6,071	12,558	4,079	16,637
35–	95,389	18,584	114,473	1,600	1,364	2,964	4,371	3,775	8,146
40–	67,380	24,576	91,956	690	1,250	1,940	1,885	3,459	5,344
45–	47,522	27,172	74,694	252	870	1,122	688	2,407	3,095
50–	37,155	34,057	71,212	96	703	799	262	1,945	2,207
55–	25,815	32,586	58,401	33	377	410	90	1,043	1,133
60–	21,649	41,725	63,374	12	263	275	33	728	761
65–	13,055	37,567	50,622	7	117	124	19	324	343
70–	9,629	39,284	48,913	1	45	46	3	124	127
75–	5,068	28,139	33,207	1	10	11	3	28	31
80 and upwards	3,111	25,453	28,564	..	6	6	..	17	17
	2,297,759	333,926	2,631,685	48,555	7,792	56,347	132,643	21,563	154,206

The women who became wives in 1851, and whose ages were given, are classified in the following table. It does not affect any of the present calculations, but may perhaps be useful in future operations.

TABLE II.

Age.	Spinsters married to Bachelors.	Spinsters married to Widowers.	Widows married to Bachelors.	Widows married to Widowers.	Total.
15–	6,699	155	5	..	6,859
20–	26,397	1,253	255	64	27,969
25–	9,337	1,281	578	222	11,418
30–	2,588	1,004	685	462	4,739
35–	764	606	429	519	2,318
40–	251	401	269	531	1,452
45–	75	189	125	425	814
50–	16	78	47	294	435
55–	4	31	20	164	219
60–	2	7	7	73	89
65–	1	1	1	19	22
70–	..	..	..	7	7
75–	..	..	..	3	3
80 and upwards }	..	1	..	2	3
	46,134	5,007	2,421 .	2,785	56,347

It may be thought that the marriages of a single year are insufficient data for the foundation of a table of probabilities, and, I confess, that it was from the convenience alone of the census of 1851, giving the exact number living in that year, that I was, in the first instance, led to adopt the marriages of that year only, as the basis of my calculations.

A reference, however, to two most interesting papers, by Mr. Samuel Brown, published in the *Assurance Magazine*, one of which, " On the Uniform Action of the Human Will," was read before this Institute in May, 1852, and the second, " On the Proportion of Marriages at different Ages of the Sexes," before the British Association, at Dublin, in August, 1857, showing how uniform are the proportions of marriages from year to year, convinced me that the results obtained from the single year 1851 would be sufficiently accurate for my present purpose.

Mr. Brown has given tables, deduced from the Registrar-General's Reports, of the proportion per cent. which the number of marriages in each of the following classes bears to the total number; and of the proportion per cent. of marriages at four periods of age. To these I have added, in the last column, the proportions for the year 1851 alone, which will be found very closely to correspond.

Proportion per Cent. of Marriages.

	1846-7-8.	1851-2-3.	1851.
Bachelors with Spinsters	83·515	82·578	81·88
Bachelors with Widows	3·735	4·211	4.30
Widowers with Spinsters ...	8·401	8·504	8·87
Widowers with Widows....	4·349	4·707	4·95
	100·000	100·000	100·00
Husbands under 30	76·770	75·624	75·56
30 to 45	18·310	19·219	19·49
45 to 60	4·031	4·248	4·13
60 and upwards ..	·889	·909	·82
	100·000	100·000	100·00

I am confirmed in my opinion of the sufficiency of the data, by a comparison which Dr. Farr has instituted between his two English Life Tables. The first was constructed from the numbers living in 1841, and the deaths in that year alone; and the second, from the population in 1841, and the deaths in the seven years, 1838 to 1844, inclusive.

AGES.	EXPECTATION OF LIFE.		VALUE OF £1 ANNUITY.	
	E. L. T., No. 1.	E. L. T., No. 2.	E. L. T., No. 1.	E. L. T., No. 2.
10	47·08	47·47	23·033	23·204
20	39·88	39·99	21·177	21·233
30	33·13	33·21	19·135	19·194
40	26·57	26·46	16·721	16·669
50	20·03	19·87	13·737	13·627
60	13·59	13·60	10·060	10·057
70	8·52	8·55	6·652	6·684

The differences between these tables, founded on deaths, being so trifling, it may reasonably be anticipated that results founded on marriages would bear an equally favourable comparison; in support of which I may adduce the testimony of M. Quetelet, who has shown that in Belgium, during 20 years, the extreme variation in the total number of marriages, from year to year, was little more than half the difference of the extremes in the number of deaths in the same period.

As the event by which the Insurance Company would become liable, could only occur through marriage with a woman of child-

bearing age, it had been my original intention to exclude those marriages where the age of the wife was above 50 years, but finding that they amounted to less than 1½ per cent. on the total number, and looking with some slight suspicion on the ages of ladies as given in marriage certificates, I considered it would be safer to take the probability of marrying at all, regardless of the age of the wife.

The following table of the probability of bachelors, widowers, and of the two combined, marrying in the year, has been constructed from Table No. 1.

Age.	Probability of Marrying in a Year.		
	Bachelors.	Widowers.	Bachelors and Widowers combined.
20–	·11209	·30766	·11276
25–	·12209	·35791	·12826
30–	·07851	·28627	·09550
35–	·04558	·20313	·07116
40–	·02798	·14075	·05811
45–	·01448	·08858	·04144
50–	·00705	·05711	·03099
55–	·00349	·03201	·01940
60–	·00152	·01745	·01201
65–	·00146	·00862	·00677
70–	·00031	·00316	·00260
75–	·00059	·00100	·00093
80–	·00000	·00067	·00060

This table exhibits results of a rather startling character. In the first two quinquennial periods, the probability of a widower marrying in a year is nearly three times as great as that of a bachelor. At 30 it is nearly four times as great; from 30 to 45 it is five times as great; and it increases, until at 60, the chance of a widower marrying in a year is eleven times as great as that of a bachelor. It is curious to remark, from this table, how confirmed either class becomes in its condition of life—how little likely, after a few years, is a bachelor to break through his settled habits and solitary condition; and, on the other hand, how readily in proportion does a husband contract a second marriage who has been deprived prematurely of his first partner.

After the age of 30, the probability of a bachelor marrying in a year diminishes in a most rapid ratio. The probability at 35 is not much more than half that at 30, and nearly the same proportion exists between each quinquennial period afterwards. Many

curious speculations as to the social condition might be founded on these results, upon which, however, it is not my province to enter.

The following table, showing the probability of a widower of every age above 40 marrying in a year, has been derived from the previous results by the method of interpolation by third differences —lives under 40 years of age having been omitted as seldom coming practically before us, and the marriages in each quinquennial period, according to Dr. Farr's tables, having been taken to represent the marriages at the youngest age of that period.

PROBABILITY OF A WIDOWER MARRYING IN A YEAR.							
40	·14075	50	·05711	60	·01745	70	·00316
41	·12800	51	·05107	61	·01542	71	·00246
42	·11652	52	·04557	62	·01352	72	·00190
43	·10620	53	·04058	63	·01175	73	·00147
44	·09693	54	·03607	64	·01011	74	·00117
45	·08858	55	·03201	65	·00862	75	·00100
46	·08197	56	·02852	66	·00721	76	·00085
47	·07549	57	·02534	67	·00597	77	·00074
48	·06197	58	·02245	68	·00489	78	·00068
49	·06304	59	·01983	69	·00396	79	·00066
						80	·00067

There is at present hardly sufficient data from which to calculate the probability of issue from second marriages, and that part of the question must still be solved by conjecture; but, as it will be allowed that one of the principal items composing the risk to be borne by Assurance Societies will be that of the husband contracting a second marriage, the theory which I now propose is the assumption that the amount assured will absolutely become payable on the happening of that event. Rough as this approximation appears, the resulting premiums are still very much below what may be considered the present market price.

Assuming, then, that husband and wife are both alive, and that issue by the present marriage is impossible, the payment of the sum assured at the end of the nth year will depend—1, on the wife having died in or before the nth year; 2, on the husband surviving; and 3, on the widower remarrying in the year. The expression for the value of the payment at the end of the nth year will, therefore, be

$$p_{\text{H},n}(1-p_{\text{W},n})\phi_{\overline{\text{H}+n-1}}r^n,$$

H and W representing the respective ages of husband and wife, and ϕ the probability of a widower marrying in a year.

The summation of these terms for every year will give the single premium for an assurance payable whenever the second

marriage takes place. The formula is not strictly accurate, as the probability of the wife dying extends to the end of the nth year, and the probability of the marriage of the widower cannot commence until after the death of the wife. The error is on the safe side, and is, I believe, immaterial.

The following table has been constructed from the probabilities of living derived from the English Life Table (No. 1), males and females, and 3 per cent. is the rate of interest assumed.

Present Value of £1 payable at the end of the Year in which a Husband shall contract a Second Marriage. English Life Table. Interest 3 per Cent.

AGE OF HUSBAND.	AGE OF WIFE.								
	40.	45.	50.	55.	60.	65.	70.	75.	80.
40	·09090	·10414	·12676	·16951					
45	·04928	·05595	·06940	·09090	·12482				
50	·02452	·02760	·03283	·04458	·06197	·08361			
55	·01098	·01226	·01450	·01959	·02757	·03798	·05110		
60			·00559	·00757	·01075	·01502	·02056	·02738	
65			·00181	·00241	·00345	·00490	·00679	·00922	
70			·00046	·00063	·00089	·00130	·00184	·00250	·00337
75					·00020	·00029	·00041	·00058	·00081

The calculations were made by aid of the perforated cards first introduced by Mr. Peter Gray—for which, and for the numerous other methods he has invented for the diminution of labour in the construction of tables, actuaries and computers are infinitely indebted. The expression given above is divisible into two parts, the one relating to the husband and the other to the wife; and for each a card has been prepared, as in the following diagram, representing the logarithms of the probabilities at each year of age. A husband of any given age may thus be combined with a wife of any other.

By combining the cards, according to the diagram, and by a simple addition of the quantities for each year throughout, as in the example, a column will be formed, upon removing the cards, representing the logarithms of the present value of the payment in each year. The natural numbers are then to be found, the sum of which will be the single premium for the risk. I should mention that, in this operation, I have used Professor De Morgan's card of four figure logarithms, the use of which would, I feel sure, be far more general were its existence better known.

3.

W 50.

H 50.
$\overline{5}\cdot7007$
$\overline{1}\cdot8932$
$\overline{5}\cdot5939$
$\overline{5}\cdot7871$
$\overline{1}\cdot8763$
$\overline{5}\cdot6634$
$\overline{5}\cdot8927$
$\overline{1}\cdot8579$
$\overline{5}\cdot7506$
$\overline{4}\cdot0175$
$\overline{1}\cdot8379$
$\overline{5}\cdot8554$

$\lambda p_{50,\,30}\,r^{30}\phi_{79}$
$\lambda(1-p_{50,\,30})$

2.

W 50.

$\overline{1}\cdot8932$
$\overline{1}\cdot8763$
$\overline{1}\cdot8579$
$\overline{1}\cdot8379$

$\lambda(1-p_{50,\,30})$
$\lambda(1-p_{50,\,29})$
$\lambda(1-p_{50,\,28})$
$\lambda(1-p_{50,\,27})$

1.

H 50.
$\overline{5}\cdot7007$
$\overline{5}\cdot7871$
$\overline{5}\cdot8927$
$\overline{4}\cdot0175$
$\overline{4}\cdot1489$

$\lambda p_{50,\,30}\,r^{30}\phi_{79}$
$\lambda p_{50,\,29}\,r^{29}\phi_{78}$
$\lambda p_{50,\,28}\,r^{28}\phi_{77}$
$\lambda p_{50,\,27}\,r^{27}\phi_{76}$
$\lambda p_{50,\,26}\,r^{26}\phi_{75}$

The net premiums thus obtained would be loaded according to the discretion of the actuary employing them; and as it is an established rule that all special premiums should be taxed in this respect more severely than others, I have assumed in the following comparisons an addition of 100 per cent. The examples given are cases in which large reassurances have been effected, and the premiums may, therefore, be considered to have had the approval of more than one actuary.

EXAMPLE I.

Husband aged 48, wife 54. Sum payable on death of husband in the event of issue sur viving, or having attained the age of 21 in lifetime of the father. Single premium charged   .   .   .   .   .   .   £14   0   0 per cent.

Premium for a sum payable in the event of the second marriage of husband   .   .   .   11 19   0   „

EXAMPLE II.

Husband 60, wife 56. Sum payable on death of husband, in the event of male issue sur viving. Single premium charged   .   .   £10   0   0   „

Single premium for a sum payable in the event of second marriage of husband .   ˙.   .   1 13   0   „

EXAMPLE III.

Husband 78, wife 63. Sum payable at death of husband, in the event of male issue sur viving. Single premium charged   .   .   £5   5   0   „

Single premium for a sum payable in the event of second marriage of husband   .   .   { A merely nominal sum (about 1s.)

Although these differences border upon the marvellous, and the new rates will probably be deemed impracticable, I cannot but think they should be sufficient. The objections urged would probably be :—1. That the numbers in this class of insurances would be too small to form an average. 2. That the probabilities of marriage among the class of persons to which these risks are restricted would, perhaps, be greater than those of the general population. 3. That there would be greater liability to fraud in these than in ordinary insurances.

Upon the other hand, the suggested premiums assume that the sum assured would become payable on re-marriage, independent of issue; and they have further been subjected to an exceptional loading, surely sufficient to counterbalance the above objections.

I have little doubt that the probabilities of marriage among the

higher classes would be found to be greater than those obtained from the general population, as among such there would naturally be anxiety for issue—to preserve titles from extinction, or to keep landed estates in particular families. It is probable, too, that the personal feeling which occasionally exists on the part of the tenant for life against his successor, would tend to increase the probabilities of marriage; and, in reference to these points, it has been suggested to me, that the Peerage and Baronetage afford abundant materials from which satisfactory conclusions might be drawn. Many other interesting investigations might be undertaken from the collection of facts there contained, the results of which might be speedily rendered available by a few members of the Institute acting in combination.

The risk of fraud in these transactions, I am disposed to think, is generally exaggerated, assuming that reasonable inquiry is made when the insurance is granted. The only attempt which has come to my knowledge was in the case of a person who married a woman already pregnant, and fortunately this failed, the wife having miscarried.

Hitherto I have treated only of the question of single premiums. The annual premium, being payable during the life of the husband, should theoretically be derived from the single premium by the usual mode of dividing by 1 + the value of an annuity of £1 during his life; and if it were certain that the insurance would be kept up till the end of life, there could be no objection to this method; but as in contradistinction to ordinary insurances, the chief portion of the risk occurs in the earlier years, it cannot practically be adopted, since the policy would most probably be dropped when the risk was much diminished by increased age. It is, therefore, always desirable that these policies should be effected by a single payment.

In conclusion, I trust that although the foregoing observations may be considered crude, and the mode of approximation suggested not always practicable, I have nevertheless opened a new subject of discussion for the Institute, and have at least aided in removing any reproach which might be considered as attaching to the younger members of the Institute for contributing as yet but little to the literature of it.